Exercise 3: Let X1, X2, X3, X4 be a sample from U(0, 1), and let X(1), X(2), X(3), X(4) be the order statistic. Determine the density of X(1), X(2), X(3), X(4).

Solution:

For *X1, X2, ..., Xn* iid continuous random variables with pdf *f* and cdf *F* the density of the *kth* order statistic is

*f*k(x) dx = *P*(X(k) € dx)

= *P* (One of the X’s € dx, k -1 of the others < x)

= n \* *P* (X1 € dx, (k-1) others (exactly) < x)

= n \* *P* (X1 € dx) (( (F(x)) (k-1) \* (1 – F(x)) (n-k))

= n \* *f(x)* dx \* (( \* (F(x)) (k-1) \* (1 – F(x)) (n-k))

= \* *f(x)* dx\* (F(x)) (k-1) \* (1 – F(x)) (n-k))

Let X1, X2, ..., Xn U(0,1) then the density of X(n) is given by [1]:

*f*k(x) = \* *f(x)* \* (F(x)) (k-1) \* (1 – F(x)) (n-k))

=

Density of X (1) = n \* (1 – x) (n – 1)

Density of X (2) = n \* (n – 1) \* x \* (1 – x) (n-2)

Density of X (3) = \* x2 \* (1 – x) (n-3)

Density of X (4) = \* x3 \* (1 – x) (n-4)

Reference:

[1] https://www2.stat.duke.edu/courses/Spring12/sta104.1/Lectures/Lec15.pdf